

# Lattice Green's functions

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## Contents

<b>1</b>	<b>Lattices and their Green's functions</b>	<b>1</b>
1.1	Definitions . . . . .	1
1.2	Fourier transform . . . . .	3
1.3	Primitive lattices . . . . .	3
1.4	Series representation . . . . .	4
1.5	Continual approximation . . . . .	4
1.6	Evaluation of lattice Green's functions . . . . .	4
1.7	Estimates and inequalities for $s \geq 0$ . . . . .	5
<b>2</b>	<b>Hypercubic lattice</b>	<b>5</b>
2.1	General statements . . . . .	5
2.2	Series at infinity . . . . .	6
2.3	Asymptotic expansion in $ x \sqrt{s}$ . . . . .	7
2.4	Large $x$ approximation . . . . .	7
<b>3</b>	<b>One-dimensional lattices</b>	<b>8</b>
<b>4</b>	<b>Square lattice</b>	<b>10</b>
<b>5</b>	<b>Simple cubic lattice</b>	<b>11</b>
5.1	Series at infinity . . . . .	11
5.2	Exact results . . . . .	11
5.3	Complex variable method . . . . .	11
5.4	Other . . . . .	12
<b>6</b>	<b>Other lattices</b>	<b>12</b>
6.1	Triangular lattice . . . . .	12
6.2	Face-centered cubic lattice . . . . .	12
6.3	Diamond lattice . . . . .	12
6.4	Body-centered cubic lattice . . . . .	13
6.5	Decorated lattices . . . . .	13
<b>7</b>	<b>Periodic simple hypercubic lattice</b>	<b>13</b>
<b>A</b>	<b>Some properties of finite difference operators on <math>\mathbb{Z}</math></b>	<b>14</b>

## 1 Lattices and their Green's functions

### 1.1 Definitions

*Graph* is a collection of points (enumerable set  $X$ ) and bonds (subset of symmetrized  $X^2$ ) with finite coordination number, which is the maximum number of bonds attached to a point (i.e. simple undirected finite-dimensional graph). In what follows we consider only connected graphs and denote a graph simply by the set of its points  $X$ . The bonding between points establishes the metrics as the length of the shortest path between two points.