Handbook on physical units and constants

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§1. Basic facts. Notations and conventions

System of units: International SI (m, kg, s, A, K, mol, cd), Symmetric CGS (cm, g, s), Electrostatic CGSE, Electromagnetic CGSM, $\hbar = c = 1$ (g), and some technical systems. In this paper we use CGS.

To change unit in an expression use the following scheme. Let we have amount X in units u^n, briefly $x = X\ u^n$. We want to transform this expression to unit v which is in the following relation to u: $1\ u = Z\ v$, briefly $u = Z\ v$. Then $X\ u^n = XZ^n\ v^n$.

We denote by $[x] \equiv X$ the value of the physical quantity x without units. If the unit system is different from CGS, it is written as subindex like this $[x]_{\mathrm{SI}}$.

The *standard uncertainty* of a scientific constant is its estimated standard deviation [1]. For example, $x = 1.234(12)$ means that the expectation of x is 1.234 and the standard deviation of x is 0.012. If x is normally distributed then with probability 68% its value belongs to the interval 1.234 ± 0.012.

Note that "g/cm s" means "g cm^{-1} s^{-1}".

Fundamental constants		
speed of light	c	$2.99792458 \cdot 10^{10}$cm/s (exact)
Planck constant	$\hbar$	$1.0545726(6) \cdot 10^{-27}$g cm^2/s $= 6.5821215 \cdot 10^{-16}$eV s
gravitation constant	G	$6.67259(85) \cdot 10^{-8}$cm^3/g s^2
fine structure constant	α	$0.00729735308(33)$ $(= e^2/\hbar c)$
Avogadro constant	N_{A}	$6.0221367(36) \cdot 10^{23}$mol^{-1}

§2. Fundamental units

Note: After quantity name its units in CGS, SI and $\hbar = c = 1$ are written.

Length (cm, m, g^{-1})				
Planck length	$l_{\rm Pl}$	$\sqrt{\frac{G}{\hbar c^3}}$	$1.62 \cdot 10^{-33}$	cm
Compton wavelength of proton	$\lambda_{\rm p}$	$\frac{2\pi\hbar}{m_{\rm p}c}$	$1.32 \cdot 10^{-13}$	cm
Compton wavelength of electron	$\lambda_{\rm e}$	$\frac{2\pi\hbar}{m_{\rm e}c}$	$2.43 \cdot 10^{-10}$	cm
Angstrom unit	A		10^{-8}	cm
Bohr radius	$a_{\rm B}$	$\frac{\hbar^2}{m_{\rm e}e^2}$	0.529	A
Earth radius	$R_\oplus$		$6.371 \cdot 10^{8}$	cm
Sun radius	$R_\odot$		$6.9599(7) \cdot 10^{10}$	cm
astronomical unit	UA		$1.49597870(2) \cdot 10^{13}$	cm
light year	ly	$c \cdot$ year	$0.9460530 \cdot 10^{18}$	cm
parsec	pc	$\frac{1"}{\pi}$UA	$3.085778 \cdot 10^{18}$	cm

Time (s, s, g^{-1})				
Planck time	$t_{\rm Pl}$	$\sqrt{\frac{G}{\hbar c^5}}$	$5.39 \cdot 70^{-14}$	s
tropical year	year	365.24 days	$3.1556922 \cdot 10^{7}$	s

Mass (g, kg, g)					
Planck mass	$m_{\rm Pl}$	$\sqrt{\frac{G}{\hbar c^3}}$	$2.18 \cdot 10^{-5}$	g	
electron mass	$m_{\rm e}$		$9.1093897(54) \cdot 10^{-28}$	g	= 0.511 MeV
proton mass	$m_{\rm p}$		$1.6726231(20) \cdot 10^{-24}$	g	= 938 MeV
atomic mass unit	$m_{\rm u}$	$[N_{\rm A}]^{-1}$	$1.660538921(73) \cdot 10^{-24}$	g	denoted also ‘u’
air molecule mass			28.964420	u	
Earth mass	$M_\oplus$		$5.976 \cdot 30^{27}$	g	
Sun mass	$M_\odot$		$1.989(4) \cdot 10^{33}$	g	

Energy (erg = g cm^2/s^2, J = kg m^2/s^2=10^7 erg, g)					
electronvolt	eV	$\frac{[e]}{[c]}10^8$	$1.60218733(44) \cdot 10^{-12}$	erg	
thermochemical calorie	cal		4.184	J	exact
specific energy	kcal/mol	$\frac{[\text{kcal/eV}]}{[N_{\rm A}]}$	0.043364	eV	
Hartree energy	hartree	$\frac{m_{\rm e}e^4}{\hbar^2}$	$2 \times 13.6056923(12)$	eV	
Rydberg energy	Ry	$\frac{m_{\rm e}e^4}{2\hbar^2}$	13.6056923(12)	eV	
Coulomb energy		e^2	14.400	eV Å	
electron confinement energy		$\frac{\hbar^2}{m_{\rm e}}$	7.6200	eV Å^2	
nuclei confinement energy		$\frac{\hbar^2}{m_{\rm u}}$	4.180	meV Å^2	
Sun luminosity	$L_\odot$		$3.846 \cdot 10^{33}$	erg/s	

Charge (esu^2 = g cm^3/s^2 ≡ erg cm, C ≡ A s = [c]/10 esu, 1)					
Ampere	A	$[c]/10$	$0.299752458 \cdot 10^{10}$	esu/s	exact
electron charge	e		$4.8032068(15) \cdot 10^{-10}$	esu	$1.60217733(49) \cdot 10^{-19}$ C
Bohr magneton	$\mu_{\rm B}$	$\frac{e\hbar}{2m_{\rm e}c}$	$9.2540654(31) \cdot 10^{-21}$	cm esu	erg/G
nuclear magneton	$\mu_{\rm N}$	$\frac{e\hbar}{2m_{\rm p}c}$	$2.0407846(17) \cdot 10^{-24}$	cm esu	erg/G
		$\mu_{\rm e}/\mu_{\rm B}$	1.002159622193(10)		
		$\mu_{\rm p}/\mu_{\rm N}$	2.722867386(63)		

§3. Mechanics

Mechanical quantities and their units				
momentum	p		kg m/s	
angular momentum	L		kg m^2/s	
moment of inertia	I		kg m^2	
force	F	N	kg m/s^2	
moment of force (torque)	M		N m	
pressure	p	Pa	kg/m s^2	bar=10^5 Pa
dynamic viscosity	η	P	g/cm s	
kinematic viscosity	ν	St	cm^2/s	
surface tension	α		N/m	
work	A		J	
power	P	W	kg m^2/s^3	

Pressure	
ultra-high vacuum	<100 nPa
ear sensitivity	20 μPa
high vacuum	<100 mPa
US dollar bill	1 Pa
ear-safe	<200 Pa
1 atm	101 kPa
human bite	1 MPa
Earth core	360 GPa
diamond anvil cell	500 GPa
Sun core	$2.5\cdot 10^{16}$ Pa

Acoustic waves ($c_{\rm air}=330\,$m/s)			
ν	$\lambda = c_{\rm air}/\nu$	octaves	
<40 Hz	>10 m		infrasound
16 Hz – 22 kHz	15 mm – 28 m	10	sound
1 – 4 kHz	10 – 34 cm	2	best sound
20 kHz – 1000 MHz	0.3 μm – 15 mm	16	ultrasound
10^9 – 10^{13} Hz	0.03 – 300 nm	13	hypersound

§4. Thermodynamics and molecular physics

Thermodynamic constants				
Boltzmann constant	k		$1.380658(02)\cdot 10^{-16}$	erg/K
molar gas constant	R	$kN_{\rm A}$	8.334510(70)	J/mol K
Stefan–Boltzmann constant	σ	$\frac{\pi^2 k^2}{60\hbar^3 c^2}$	$5.87051(19)\cdot 10^{-8}$	W/m^2K^4

Standard atmosphere		
lattitude	φ	$45°32'40''$
acceleration of gravity	g	980.665 cm/s^2
pressure	$p_{\rm A}$	101325 Pa (1 atm or 760 mm Hg)
temperature	$T_{\rm A}$	288.15 K (15°C)
molecular weight	$m_{\rm A}$	28.9644 u

Energy-temperature-wavelength-time conversion is based on the identities $E=kT$, $E=2\pi\hbar c/\lambda$, $E=2\pi\hbar/P$:

$$1 \text{ eV} = 11600 \text{ K} = (1240 \text{ nm})^{-1} = (4.136 \text{ fs})^{-1}, \quad 1000 \text{ K} = 86.17 \text{ meV}, \quad 1 \text{ cm}^{-1} = 0.124 \text{ meV}, \quad 1 \text{ fs}^{-1} = 4.136 \text{ eV}$$

Energy-pressure conversion based on the identity $E=pV$: 1 eV/Å^3=160 GPa, 1 GPa=6.24 meV/Å^3.

§5. Electromagnetism and optics

In SI there are two fundamental constants: permittivity of vacuum $\varepsilon_0 = 10^{11}/\left(4\pi[c]^2\right)$ F/m and permeability of vacuum $\mu_0 = 4\pi 10^{-7}$ H/m. Note also that all magnetic quantities in SI are in some sense multiplied by the speed of light in comparison with electrical quantities, so their dimensions differ.

Electromagnetic quantities and their units						
charge	q	C	A s	$10^{-1}[c]$	esu	
current	I	A	A	$10^{-1}[c]$	esu/s	
electric potential	U	V	kg m^2/A s^3	$10^8[c]^{-1}$	esu/cm	e·V≡ eV
electric intensity	E	V/m	kg m/A s^3	$10^6[c]^{-1}$	esu/cm^2	
electric displacement	D	C/m^2	A s/m^2	$4\pi 10^{-5}[c]$	esu/cm^2	
polarization	P	C/m^2	A s/m^2	$10^{-5}[c]$	esu/cm^2	
magnetic flux density	B	T	kg/A s^2	10^4	esu/cm^2	=G
magnetizing force	H	A/m	A/m	$4\pi 10^{-3}$	esu/cm^2	=Oe
magnetization	M	A/m	A/m	$(4\pi)^{-1}10^4$	esu/cm^2	=G
magnetic flux	Φ	Wb	kg m^2/A s^2	10^8	esu=Mx	
electric dipole moment	p	C m	A m s	$10[c]$	esu cm	debye,D=10^{-18} esu·cm, e·Å=4.8 D
magnetic moment	μ	J/T	A m^2	10^3	esu cm	
resistance	R	Ω	kg m^2/A^{2}s^3	$10^9[c]^{-2}$	s/cm	
resistivity	ρ	Ω m	kg m^3/A^{2}s^3	$10^{11}[c]^{-2}$	s	
capacitance	C	F	A^{2}s^4/kg m^2	$10^{-9}[c]^2$	cm	
inductance	L	H	kg m^2/A^{2}s^2	10^9	cm	
polarizability	α	C m^2/V	A^{2}s^2/kg	$10^{-5}[c]^2$	cm^3	

Magnetic fields	
galaxy clusters	1–2 μG
Milky Way	5 μG
near Earth	10–100 μG
Earth equator	0.35 G
Earth poles	0.65 G
Sun poles	1–2 G
human-safe	< 50 kG
white dwarfs	1–10 MG
neutron stars	10^{10}–10^{13} G

Resistivity ρ, Ω cm	
metals	$10^{-6} - 10^{-4}$
semiconductors	$10^{-3} - 10^{7}$
insulators	$10^{8} - 10^{18}$
electrolytes	$10^{0} - 10^{10}$
water, alcohols	$10^{5} - 10^{8}$
solid electrolytes	$10^{1} - 10^{3}$
Si	$2.3 \cdot 10^5$
SiO_2	10^{13}

Mobility μ, cm^2/V s $\sigma = \mu n e$, $D = \mu T/e$	
hopping	< 1
e in Si	$1.5 \cdot 10^7$
h in Si	$0.5 \cdot 10^7$
H^+ in water	$3.3 \cdot 10^{-3}$
OH^- in water	$1.8 \cdot 10^{-3}$
Na^+ in water	$0.5 \cdot 10^{-3}$
solid electrolytes	$10^{-4} - 10^{-3}$

Electromagnetic spectrum			
λ	$\nu = c/\lambda$	$E = 2\pi\hbar c/\lambda$	
RADIOWAVES (35 octaves)			
> 10 km	< 30 kHz		superlong RW
1–10 km	30–300 kHz		long RW
0.1–1 km	0.3–3 MHz		medium RW
10–100 m	3–30 MHz		short and ultrashort RW, AM radio
1–10 m	30–300 MHz	0.1–1 μeV	meter RW, FM radio
1 mm – 30 m			the first window in Earth atmosphere
21 cm			neutral hydrogen line
1–10 dm	0.3–3 GHz		ultrahigh frequencies (UVCh)
1–10 cm	3–30 GHz		microwaves, SVCh
1–10 mm	30–300 GHz	0.1–1 meV	microwaves
1 mm			2.7 K cosmic microwave background
0.1–1 mm	0.3–3 THz	1–10 meV	submillimeter RW
OPTICAL BAND (17 octaves)			
INFRARED BAND (11 octaves)			
0.05–2 mm		0.5–25 meV	far IR
50 μm		26 meV	the heat energy of molecules at 20°C
10 μm			blackbody radiation at 20°C
2.5–50 μm		25–500 meV	medium IR, L,M,N,Q bands
1.5 μm			T=1900 K blackbody radiation (candle)
1 μm			T=2900 K blackbody radiation (incandescent lamp, red stars)
0.8–2.5 μm		0.5–1.5 eV	near IR, I,J,H,K bands, Si band (1.1 eV)
VISIBLE BAND (1 octave)			
390–760 nm		1.5–3 eV	cone vision (best for 556 nm), visible band
400–650 nm			rod vision (best for 510 nm)
500 nm		2.48 eV	Sun spectral maximum (T=5780 K)
300 nm–2 μm			the second window in Earth atmosphere
700 nm		1.77 eV	R band, red color
550 nm		2.25 eV	V band, green color
440 nm		2.82 eV	B band, blue color
ULTRAVIOLET BAND (5 octaves)			
10–400 nm		3–100 eV	ultraviolet band
360 nm		3.44 eV	U band
290 nm			T=10000 K blackbody radiation (A0 stars)
X-RAYS (10 octaves)			
0.2–10 nm		0.1–5 keV	soft X-rays
0.01–0.2 nm		5–100 keV	hard X-rays
GAMMA-RAYS (40 octaves)			
$10^{-(10\div9)}$ cm		0.1–1 MeV	soft γ-rays
$10^{-(11\div10)}$ cm		1–10 MeV	medium γ-rays
$10^{-(15\div11)}$cm		10^{-2}–10^{2} GeV	hard γ-rays
$10^{-(21\div15)}$ cm		10^{-1}–10^{5} TeV	ultrahigh energy γ-rays

§6. System $\hbar = c = 1$

Physical quantities	
m^{-1}	r, t
1	v, S, e, α
m	p, E, A
m^2	F, E, B
m^4	$\mathcal{L}$

Fundamental units			
erg	c^{-2}	$1.11265 \cdot 10^{-21}$	g
eV		$1.78268 \cdot 10^{-33}$	g
cm	$c\hbar^{-1}$	$2.8427 \cdot 10^{37}$	g^{-1}
s	$c^2\hbar^{-1}$	$8.5223 \cdot 10^{47}$	g^{-1}

§7. Atomic system of units ($m_\mathrm{e} = \hbar = e = 1$)

Fundamental units		
length	a_B	$\frac{\hbar^2}{m_\mathrm{e}e^2}$
time		$\frac{\hbar^3}{m_\mathrm{e}e^4}$
mass	m_e	m_e
energy	hartree	$\frac{m_\mathrm{e}e^4}{\hbar^2}$
charge	e	e

Other units		
electric dipole moment	$e \cdot a_\mathrm{B}$	$\frac{\hbar^2}{m_\mathrm{e}e}$

References

[1] http://physics.nist.gov/cgi-bin/cuu/Info/Constants/definitions.html